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Effects of inertia on the diffusional deposition of small particles to spheres and cylinders at low Reynolds numbers

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A formalism that accounts for inertial and diffusive effects in the dynamics of a dilute gas–particle suspension is introduced. The treatment is purely deterministic away from a very thin Brownian diffusion sublayer, while, within the sublayer, inertial effects are small, permitting a near-equilibrium expansion in powers of the Stokes number (particle relaxation time divided by flow characteristic residence time). This expansion provides phenomenological expressions for the particle velocity including two terms: the standard Brownian diffusion, and an additional inertial drift velocity which is closely related to the pressure diffusion term of the Chapman–Enskog expansion. As an example, the general formalism is applied in detail to the case of Stokes flow about a sphere, and sketched for the similar case of a cylinder. Two competing mechanisms are seen to affect the total rate of particle capture by the sphere: (i) the stagnation-point region is considerably enriched in particles owing to the high compressibility of the particle phase, which leads to locally enhanced deposition; (ii) centrifugal forces tend to deplete the Brownian diffusion sublayer of particles, *reducing* diffusion rates away from the stagnation point to the surface. The first effect is seen to dominate over the second except in a very narrow zone of small Stokes numbers. Our method bridges the gap between Levich's solution for the 'pure-diffusion' limit and Michael's treatment in the 'pure-inertia' limit.

1. Introduction

We address the problem of the motion of a swarm of very small particles suspended in a host gas, in the transition region where the relaxation time τ of the particles (see (7) below) passes from being small (diffusion limit) to being comparable to or larger than the host-fluid residence time ω^{-1} (inertia-dominated limit).

In the former limit, the particles follow closely the motion of the host fluid, and the mixture evolution may be described by single-fluid equations (with standard phenomenological transport laws), very much like a gas mixture in the continuum region. This limit, from which inertial effects are absent, is seldom considered in the literature of particulate flows. In the inertia-dominated situation the particles depart considerably from the fluid streamlines, moving on their own very much like a gas in free molecular flow. In that case the dynamics of the system are often treated using the so-called 'dusty-gas' model (Marble 1970), which basically assigns different velocity, density and temperature fields to the fluid and particle phases, including their mutual interaction in the corresponding conservation equations. The host fluid is treated as Newtonian, while the particle phase is assumed not to contribute to the pressure of the mixture, thus moving deterministically under the action of the fluid-particle drag (see also Michael 1968; Michael & Norey 1968, 1970; Morsi & Alexander 1972). Accordingly, particle Brownian motion is implicitly neglected,† and diffusion is left out of the picture. An interesting particular case within this inertia-dominated limit arises when the particle mass fraction is very small. Then the fluid motion is effectively uncoupled from the particles and may be obtained independently, as in ordinary fluid-mechanics problems. Now, since the *particle* momentum-conservation equations are first order p.d.e.s, they may be integrated along the characteristics, which are clearly equivalent to the trajectories of individual particles obeying the deterministic equations of Newton. Thus the Newtonian or Lagrangian particle trajectory method turns out to be very simple, and has been the dominant technique employed in the aerosol literature for dealing with the 'pure' inertial problem. One of the most interesting results one finds in this limit for the flow of particulate suspensions about solid obstacles is the existence of a critical value of $\omega\tau$ above which the particles are unable to 'side-step' the obstacle (as the fluid does) and impact on its surface (Fuchs 1964).

Clearly, both diffusive and inertial effects are treated in much more simple terms in the limit of small particle mass fraction, and we will confine ourselves to this limit for the study of the 'transition' region. That is, the region where $\omega\tau$ is of order unity. Generalization of these ideas to apply away from the small-particle mass-fraction limit does not present any conceptual difficulty. However, the fully coupled equations for the two phases, including their mutual interaction, are so complicated that even when diffusion is neglected, in the isothermal case with an incompressible host fluid, they have been solved only for simple homogeneous unidimensional or parallel flows (Michael 1963; Nag, Jana & Datta 1979; Peddieson 1976; Singleton 1965), and the exceptional case of the motion of a dusty gas induced by the uniform rotation of an infinite disk (Zung 1969). Other valuable solutions have been confined either to this limit of very small particle loading, or the limit of very small particle relaxation time τ (compared with macroscopic times), which is unfortunately incapable of predicting some of the interesting inertial effects that concern us here. Under these conditions it would be premature to address the problem of diffusion effects in the case of non-negligible particulate mass loading.

2. General formalism for the interaction of inertia and diffusion

2.1. *Previous work*

During the last decade several works have been devoted to the simultaneous effects of diffusion and inertia (Lee 1977; Lee & Gieseke 1979; Stechkina, Kirsh & Fuchs 1970; Yuu & Jotaki 1978). Nonetheless, the interaction of those mechanisms is still so scarcely understood that even the governing equations used in the literature are often inconsistent. For example, the following mass-conservation equation was used by Yuu & Jotaki (1978), Yeh & Liu (1974*a*), Thuan (1974), Thuan & Andres (1979) and others:‡

$$\nabla\cdot(\rho_p \mathbf{V}_p - D\nabla\rho_p) = 0. \tag{1}$$

Here ρ_p and $\mathbf{V}_p$ are the particle phase density and velocity fields, and D is the particle

† Otherwise the partial pressure of the particle phase would not be zero, since pressure is due precisely to the thermal (or Brownian) motion.

‡ Soo (1967) uses a curious variation of (1) that adds another term (see p. 38 of his book) to the Fick diffusion flux.

diffusion coefficient in the bath. Nonetheless, the correct conservation equation is not (1) but

$$\nabla . (\rho_p \mathbf{V}_p) = 0. \tag{2}$$

The addition of the diffusion flux $-D\nabla\rho_p$ is justifiable only when the mean mixture velocity $\mathbf{V}$ is used instead of $\mathbf{V}_p$. Then the mass-conservation law is written as

$$\nabla . (\rho_p \mathbf{V} - D\nabla\rho_p) = 0. \tag{3}$$

But this new expression is incapable of describing the interaction phenomena that interest us here because it does not include inertial effects. On the other hand, the correct equation (2) requires prior knowledge of the particle velocity field $\mathbf{V}_p$ (independently of $\mathbf{V}$), which is generally obtained by calculating particle Newtonian trajectories (Fuchs 1964; Fernández de la Mora & Rosner, 1981). But such deterministic calculations neglect Brownian motion; hence diffusion is left out, and this formalism is also unfit for our purposes. Since the hybrid approach of (1) provides a smooth transition between both extremes, while the phenomenological approach (3) fails to account for inertia, and diffusion is absent from the Newtonian method, it is not surprising that (1) has survived uncriticized for some time. However, two roads are now open to formulate the problem correctly: inertial effects may be included in the phenomenological approach, or the Brownian motion may be added to the Newtonian formalism. As shown below (see also Fernández de la Mora 1982), particle 'inertial drift' and 'pressure diffusion' are equivalent phenomena for host-fluid deceleration times ω^{-1} that are large compared with the particle relaxation ('stopping') time τ. Thus, since pressure diffusion is well accounted for within the phenomenological framework, so is inertia. On the other hand, to include the effect of Brownian motion within $\mathbf{V}_p$, the particle-phase momentum-conservation equation has to be written as a partial differential equation in Eulerian continuum coordinates, *retaining* the particle-phase partial-pressure tensor $\mathbf{P}_p$, which accounts for the momentum transfer due to the random (Brownian) motion. Not too far from equilibrium (i.e. at small values of $\omega\tau \equiv$ inertia parameter), this tensor is isotropic and equal to the particle-phase equilibrium osmotic pressure ((6) below). Thus the problem is closed, obeying the following set of steady-state conservation equations (Robinson 1956; Marble 1970):

$$(\mathbf{V}_p . \nabla)\mathbf{V}_p + (\mathbf{V}_p - \mathbf{V})/\tau + \rho_p^{-1}\nabla . \mathbf{P}_p = 0, \tag{4}$$

$$\nabla . (\rho_p \mathbf{V}_p) = 0, \tag{5}$$

$$\mathbf{P}_p \approx \mathbf{I}\rho_p kT/m_p. \tag{6}$$

Here τ is the particle relaxation time appearing in the following expression for the individual particle drag force due to its motion with respect to the bath:

$$\mathbf{F}_p = -m_p(\mathbf{V}_p - \mathbf{V})/\tau, \tag{7}$$

m_p is the particle mass, k is the Boltzmann constant, T the absolute temperature and $\mathbf{I}$ the unit tensor. Using Einstein's (1908) relation

$$D = kT\tau/m_p, \tag{8}$$

to first order in τ (4) gives

$$\mathbf{V}_p = \mathbf{V} - D(\nabla \ln \rho_p) - \tau(\mathbf{V} . \nabla)\mathbf{V}, \tag{9}$$

where the second (Fick diffusion) term in the right-hand side originates from the particle partial pressure† and the third term is the particle inertial drift, equal to the phenomenological pressure diffusion term, as may be directly obtained from the bath momentum-conservation equation (note the use of the pressure tensor rather than the scalar pressure):

$$\rho(\mathbf{V}\,.\,\nabla)\,\mathbf{V} = -\nabla\,.\,\mathbf{P}, \tag{10}$$

where ρ is the bath density and $\mathbf{P}$ its pressure tensor.

Thus the phenomenological approach with pressure (tensor) diffusion and the particle momentum-conservation approach with equilibrium osmotic pressure are equivalent to first order in τ, both being applicable only at small values of the inertia parameter $\omega\tau$ (compared with the critical value $(\omega\tau)_c$ above which 'pure' inertial deposition sets in (Fuchs 1964)). For values of the inertia parameter that are not small compared with the critical, the phenomenological approach, being a first-order approximation, breaks down. At the same time, the mass and momentum conservation equations (4) and (5) are still applicable, but the pressure tensor departs appreciably from the equilibrium value given by (6). Thus there is no simple way to account for inertia and diffusion within a hydrodynamical framework, and a kinetic approach is necessary (see Fernández de la Mora 1982).

2.2. *The limit of very small particle diffusivity*

In a dilute binary mixture one can define the Schmidt number Sc as the ratio of the carrier-gas kinematic diffusivity ν to the binary diffusion coefficient D ($Sc \equiv \nu/D$). Here the fact that Sc is a very large number for Brownian particles is exploited to solve the problem at the hydrodynamic level. We shall see that such a treatment is valid only in the region of subcritical values of the Stokes number. For supercritical values, particles reach the wall with a finite speed dictated by purely deterministic phenomena, and diffusion plays a negligible role. Therefore the subcritical zone, in which the problem is tractable analytically, is really the principal one where the interaction between inertia and diffusion is of interest. Accordingly, we confine ourselves to subcritical Stokes number.

Making (9) non-dimensional with the characteristic velocity U_∞ and length R, the relaxation time τ in the last term becomes the Stokes number $St = \tau U_\infty/R$, and it might seem that the equation is an asymptotic expansion in powers of St, apparently limiting the validity of any results based on (9) to the region $St \ll 1$. Such a conclusion is, however, too restrictive because in the region close to the wall the *local* value of the characteristic fluid inverse deceleration time ω is not U_∞/R, but a much smaller value associated with the local value of the tensor $\nabla\mathbf{V}$ (say, its largest (absolute-value) eigenvalue). It can be shown that, owing to fluid incompressibility and the wall boundary condition, the local Stokes number so formed tends to zero as the wall is approached. The proof is particularly simple for the three situations of parallel, rotating or stagnation-point flows, and will be sketched here.‡ For the latter case, the only relevant component of the fluid velocity is that normal to the wall, which drops to zero as the square of the wall distance y. In this case $\omega = du/dy$ and vanishes as the first power of y. For a parallel (Couette) flow, $\mathbf{V}\,.\,\nabla\mathbf{V}$ is identically zero. Finally, for a rotating fluid, ω is the ratio between the tangential velocity and the local radius of curvature, falling also to zero as y. Therefore the local value of the group $\omega\tau$

† The relation between partial-pressure gradients and Brownian (or Fick) diffusion was pointed out by Einstein (1908) (see also Frank-Kamanetskii 1969; Truesdell 1962; Chapman & Cowling 1970) and provides an elementary theory of diffusion far more useful than the more popular mean-free-path argument.

‡ For a more general discussion see Fernández de la Mora (1980, chap. 3).

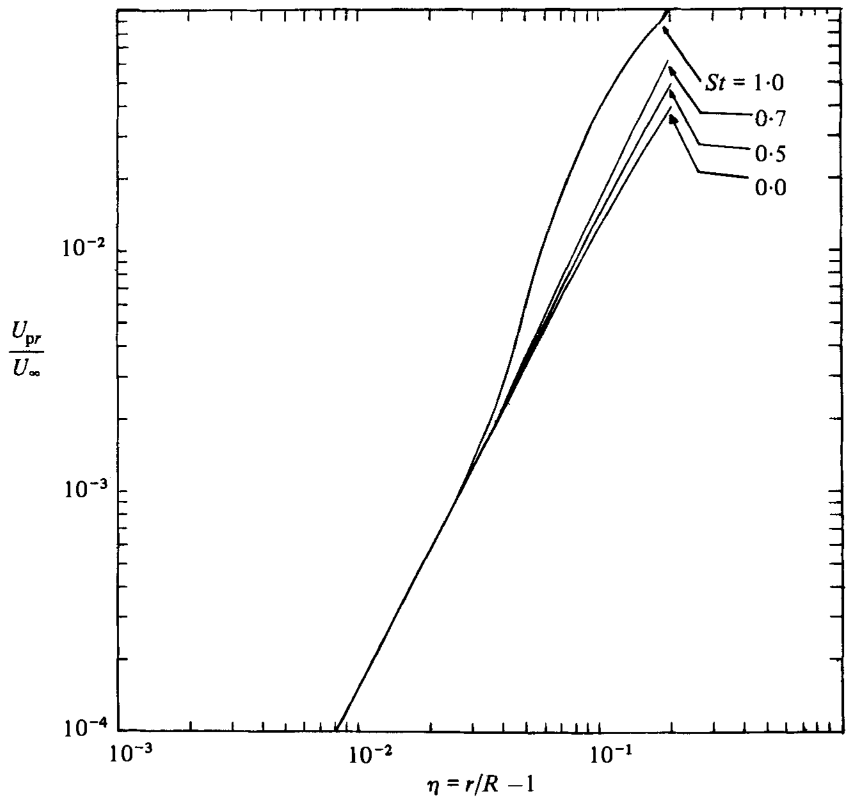

FIGURE 1. Fluid and particle velocities ($Re \ll 1$) normal to the sphere along the stagnation line as a function of non-dimensional distance to the wall. Brownian diffusion has been neglected. The numerical calculation demonstrates that for subcritical Stokes numbers there is a finite near-wall region where the fluid and particle velocity fields merge together. The Brownian-diffusion sublayer is well inside that region when $Pe \gg 1$, and consequently the simultaneous effects of inertia and diffusion are accurately described by (9) whenever $St < St_c$.

becomes arbitrarily small as the wall is approached. Furthermore, owing to the large value of the Schmidt number, the Brownian-diffusion sublayer itself lies very close to the wall, where inertia may be treated as a perturbation. Accordingly, in the region where inertia and diffusion can coexist, (9) applies (for the full range of subcritical Stokes numbers), and the problem of their interaction is formally closed. Our interpretation of (9) as an expansion in powers of the local gradients rather than the overall Stokes number requires further clarification. For the present, our justification will depend on general physical reasoning reinforced by numerical calculations. In the first place, we have already seen that the inertial term $\tau(\mathbf{V}\,.\,\nabla)\mathbf{V}$ in (9) is closely related to pressure diffusion in the Chapman–Enskog diffusion equations. But the Chapman–Enskog method is an expansion in powers of the local gradients (Chapman & Cowling 1970), in agreement with our interpretation. This point can also be verified numerically. Ignoring diffusion, one can calculate particle velocities along their Newtonian trajectories and check that at subcritical values of the overall Stokes number they closely approach (9) near the wall, and that this convergence occurs sufficiently far from the wall so that diffusion effects are indeed negligible. Results of this type may be seen in figure 4 of Fernández de la Mora & Rosner (1981) for the viscous stagnation-point flow (see also Fernández de la Mora 1980, p. 63). This agreement remains excellent up to the critical Stokes number $\omega\tau = \frac{1}{4}$. A similar behaviour can be observed in figure 1 for the low-Reynolds-number flow of a dusty gas around a sphere. For this type of flow, (9) is very inaccurate in most of the flow field, but holds true close to the wall for subcritical Stokes numbers (see also Fernández de la Mora 1980, §§2.4, 3.6, 7.6). Accordingly, the particle flow field has three distinct regions: (i) an outer one dominated by inertia where diffusion is negligible and the traditional method of neglecting the particle pressure tensor can be used to obtain the particle velocity and concentration fields; (ii) an intermediate region close to the walls (but still away from the diffusion sublayer) where Brownian motion is negligible, yet inertia can be treated as a perturbation $[\mathbf{V}_p = \mathbf{V} - \tau(\mathbf{V}\,.\,\nabla)\,\mathbf{V}]$; and (iii) the Brownian-diffusion sublayer, very close to the wall, where the convective velocity (tending to zero) becomes comparable with the particle thermal velocity. Here, both inertia and diffusion are important, and their simultaneous effects are described by (9).

3. Application to low-Reynolds-number flow about a sphere†

3.1. *Background*

Inertial effects at low Reynolds numbers often play an important role in gas filtration. The present section is, however, not directly concerned with filtration problems in which the flow field may be strongly affected by rarefaction or interference between the various filter fibres. We will simply apply the method sketched in §2 to account for the effects of inertia and diffusion on the deposition of particles to an isolated sphere moving at small Reynolds' numbers in an undisturbed fluid. The generalization to other geometries (e.g. a cylinder, which we treat in appendix B) or other flow fields (e.g. the many models used in filtration (Spielman 1977)) is straightforward.

The problem of small-Reynolds-number *diffusion* of particles to a sphere of radius R moving in an undisturbed fluid with relative velocity U_∞ was solved by Levich (1962) in the absence of inertia. Levich used the stream function as an independent variable (von Mises' method) to obtain an analytical solution to the mass-transfer problem in the limit of high Péclet numbers ($Pe = U_\infty R/D$). In this section we will generalize Levich's approach to include inertial effects in the motion of the particles. Here the role of the stream function will be played by a variable ξ which is constant along the subcharacteristics (see Cole 1968, p. 123) or particle trajectories obtained by neglecting the small second-order (diffusion) terms in the particle-phase mass-conservation equation.

3.2. *Host-fluid velocity field*

If the mass fraction of the particles in the suspension is much smaller than unity, the host fluid velocity field is negligibly disturbed, being described by Stokes' expressions for the viscous flow about a sphere (see e.g. Landau & Lifshitz 1971). Defining the new non-dimensional distance to the wall

$$\eta \equiv \frac{r}{R} - 1 \tag{11}$$

and keeping only terms of the lowest order in η (as η is negligibly small within the

† The analogous problem of a cylinder in low-Reynolds-number flow is sketched in appendix B.

Brownian diffusion sublayer), the bath velocity field in spherical coordinates (r, θ) reduces to

$$U_r = -\tfrac{3}{2}U_\infty \eta^2 \cos\theta, \tag{12}$$

$$U_\theta = \tfrac{3}{2}U_\infty \eta \sin\theta. \tag{13}$$

3.3. *Particle velocity and density fields*

Using (9) and writing the convective term $(\mathbf{V}\,.\,\nabla)\,\mathbf{V}$ in spherical coordinates, we obtain

$$u_{\mathrm{p}r} = U_r - D\frac{\partial \ln \rho_\mathrm{p}}{\partial r} - \tau\left[U_r\frac{\partial U_r}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_r}{\partial \theta} - \frac{U_\theta^2}{r}\right], \tag{14}$$

$$u_{\mathrm{p}\theta} = U_\theta - \frac{D}{r}\frac{\partial \ln \rho_\mathrm{p}}{\partial \theta} - \tau\left[U_r\frac{\partial U_\theta}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_\theta}{\partial \theta} + \frac{U_r U_\theta}{r}\right], \tag{15}$$

$$u_{\mathrm{p}\phi} = 0. \tag{16}$$

Using (12) and (13), after neglecting terms of higher order in η we find

$$\frac{u_{\mathrm{p}r}}{U_\infty} = -\tfrac{3}{2}\eta^2\cos\theta - Pe^{-1}\frac{\partial \ln \rho_\mathrm{p}}{\partial \eta} + \frac{9St}{4}\eta^2\sin^2\theta, \tag{17}$$

$$\frac{u_{\mathrm{p}\theta}}{U_\infty} = \tfrac{3}{2}\eta\sin\theta - Pe^{-1}\frac{\partial \ln \rho_\mathrm{p}}{\partial \theta}. \tag{18}$$

The particle-phase mass-conservation equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho_\mathrm{p}u_{\mathrm{p}r}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho_\mathrm{p}u_{\mathrm{p}\theta}\sin\theta) = 0 \tag{19}$$

becomes, after neglecting diffusion in the θ-direction,

$$Pe^{-1}\left(\frac{\partial^2\rho_\mathrm{p}}{\partial\eta^2} + 2\frac{\partial\rho_\mathrm{p}}{\partial\eta}\right) + \frac{\partial\rho_\mathrm{p}}{\partial\eta}\left(\tfrac{3}{2}\cos\theta - \tfrac{9}{4}St\sin^2\theta\right)\eta^2 - \tfrac{3}{2}\eta\sin\theta\frac{\partial\rho_\mathrm{p}}{\partial\theta} = \rho_\mathrm{p}\tfrac{9}{2}St\,\eta\sin^2\theta. \tag{20}$$

In the limit $St = 0$ we recover Levich's (1962) equation. Inertia introduces two different new terms: A convective motion away from the sphere (centrifugal drift), and a sink term appearing at the right-hand side of (20). This term arises as a result of the non-zero divergence of the $\mathbf{V}_\mathrm{p}$ field, and implies that the particle phase is compressible in spite of the incompressible character of the host fluid (Robinson 1956). As expected in a first-order expansion, those two terms are proportional to St.

Outer solution. Since $Pe^{-1} \ll 1$ we may neglect the diffusion terms in (20), which becomes a first-order partial differential equation. Its characteristics, called the 'subcharacteristics' of the problem (see Cole 1968, p. 123), obey the system

$$\frac{4d\eta}{\eta^2(6\cos\theta - 9St\sin^2\theta)} = \frac{-2d\theta}{3\eta\sin\theta} = \frac{2d\rho_\mathrm{p}}{9St\eta\rho_\mathrm{p}\sin^2\theta}. \tag{21}$$

Integrating, the subcharacteristics are

$$\eta = \xi\arcsin\theta\exp\left(-\tfrac{3}{2}St\cos\theta\right), \tag{22}$$

and along each of them the particle-phase density varies as

$$\rho_\mathrm{p} = n\exp\left(3St\cos\theta\right), \tag{23}$$

where ξ and n are constants for each subcharacteristic. Accordingly, ξ may be used as a new independent variable with benefits equal to those obtained via the stream-function (von Mises') method.† Analogously, it is also convenient to use n rather than ρ_p as the dependent variable. Thus we define the new variables

$$n \equiv \rho_p \exp(-3St \cos\theta) \tag{24}$$

$$\xi \equiv \eta \sin\theta \exp[\tfrac{3}{2}St \cos\theta], \tag{25}$$

leaving θ unchanged.

Boundary layer. This reduces the particle-phase mass-conservation equation to

$$\xi^{-1}\frac{\partial^2 n}{\partial \xi^2} = \frac{3Pe}{2\sin^2\theta}\exp(-\tfrac{9}{2}St\cos\theta)\frac{\partial n}{\partial \theta}.$$

Observing now that the coefficient on the right-hand side of this equation is a function of θ only, we may define the new variable

$$s \equiv \int_0^\theta \sin^2\theta' \exp(\tfrac{9}{2}St\cos\theta')\,d\theta', \tag{26}$$

leading to the familiar Leveque equation (see e.g. Stewart 1977)

$$\xi^{-1}\frac{\partial^2 n}{\partial \xi^2} = \tfrac{3}{2}Pe\frac{\partial n}{\partial s}. \tag{27}$$

Boundary conditions. The standard boundary condition *at the wall* accounting for the finite diameter d_p of the particles is $n = 0$ at $r = R + \frac{1}{2}d_p$. This condition introduces the so-called interception parameter d_p/R into the problem. For point particles ($d_p/R \to 0$), the differential equation and the wall boundary condition are compatible with solutions depending only on the similarity variable $\mu \sim \xi/s^{\frac{1}{3}}$, thus we consider explicitly the limit $d_p/R \to 0$. Finite particle size (treated, in the absence of inertia, by Friedlander 1977) unfortunately breaks the self-similarity, and therefore requires the solution of (27) as a partial differential equation. The generalization of our method to include interception is, nonetheless, conceptually solved.

In general, the *outer boundary condition* for n comes from the matching requirement with the outer (deterministic) problem. This is best solved along the characteristics (the particle Lagrangian trajectories; cf. Fernández de la Mora & Rosner 1981), and provides a value of ρ_p (or n) for each characteristic, that is, a condition of the form $n_\infty = n_\infty(\xi)$. Recall that, in the matching region (ii), n is constant along each characteristic (fixed by a value of ξ (ξ = constant). Now, the smallness of the diffusivity of particles can be exploited again to simplify the outer boundary condition to a form compatible also with the existence of self-similar solutions $n = n(\mu)$. Roughly speaking, the thickness of the diffusion sublayer is very small and only those trajectories passing very close to the wall supply material able to reach it. But those trajectories are necessarily very close to the stagnation-point trajectory, thus we anticipate that to a first approximation n_∞ will be independent of ξ, and equal to $n_\infty(0)$. In other words, only particles originating in a very thin streamtube centred about the stagnation line pass close enough to the sphere to diffuse to it. Thus, along the deterministic portion of the path their fates are so much alike that n_∞ has virtually the same value $n_\infty(0)$ for all such trajectories. For the moment we take n_∞ as a known constant (see appendix A) and write the outer boundary condition as

$$n \to n_\infty \quad (\xi \gg 1).$$

† Note that both ξ and the stream function are constant along particle streamlines.

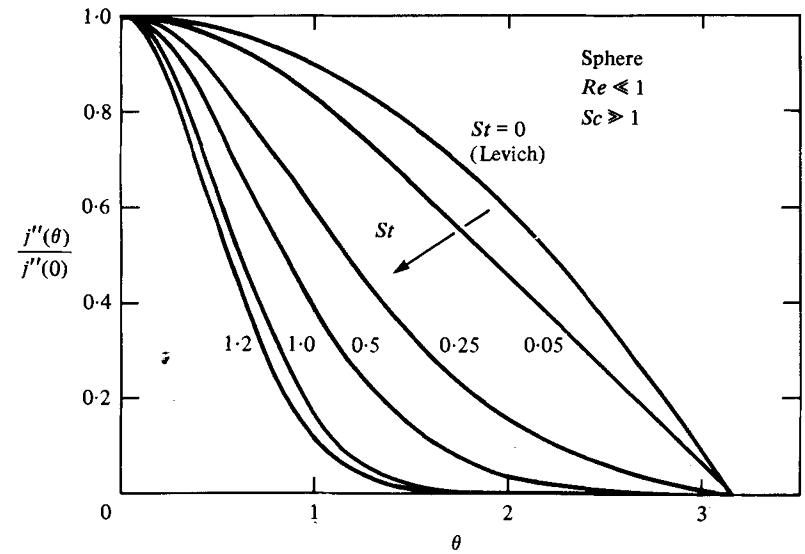

FIGURE 2. Effect of particle inertia on their diffusion to a sphere at low Reynolds number. Local deposition rates (normalized by the forward stagnation-point value (at $\theta = 0$)) *vs.* angle θ ($0 \leqslant \theta \leqslant \pi$).

The rigorous demonstration of this point, to the lowest order in an expansion in powers of D, is left to appendix C.

Solution. The solution of (27) with the boundary condition $n = 0$ at $\xi = 0$ and $n = n_\infty$ at $\xi \gg 1$ is familiar:

$$\frac{n}{n_\infty} = \frac{\int_0^\mu \exp\left(-\tfrac{1}{9}x^3\right) dx}{\int_0^\infty \exp\left(-\tfrac{1}{9}x^3\right) dx}, \tag{27'}$$

with the similarity variable μ defined as

$$\mu \equiv (\tfrac{3}{2}Pe)^{\frac{1}{3}}\xi s^{-\frac{1}{3}}. \tag{27''}$$

The wall slope is $$\left.\frac{\partial \rho_p}{\partial \eta}\right|_{\eta=0} = (\tfrac{3}{2}Pe)^{\frac{1}{3}} \frac{n_\infty 9^{-\frac{1}{3}}}{\Gamma(\frac{4}{3})} \psi(\theta, St), \tag{28}$$

with the new function $\psi(\theta; St)$ given by

$$\psi(\theta, St) \equiv \sin\theta \exp\left(\tfrac{9}{2}St\cos\theta\right)\left[\int_0^\theta \sin^2\theta' \exp\left(\tfrac{9}{2}St\cos\theta'\right) d\theta'\right]^{-\frac{1}{3}}. \tag{29}$$

n_∞ is given by (24) when ρ_p is the stagnation-point value at the outer edge of the Brownian-diffusion sublayer, which has to be calculated independently taking account of the compressibility of the particle phase (see §3.4).

The deposition rate normalized by the stagnation-point value varies as $\psi(\theta, St)/\psi(0, St)$, and is shown in figure 2 for several values of the Stokes number (note that $\psi(0, St) = 3^{\frac{1}{3}} \exp(3St)$). In the case $St = 0$ the integral appearing in (29) has an analytical expression, and the corresponding solution coincides with the one given by Levich (1962). For larger values of the Stokes number the rate of particle collection falls rapidly with distance from the stagnation point, owing to centrifugation of the particles away from the sphere.

From (28) we may obtain the local particle mass-transfer Stanton number

$$Sn_{\mathrm{m}} = \frac{j''}{\rho_{\mathrm{p}\infty} U_\infty} = \frac{Pe^{-\frac{2}{3}}}{2^{\frac{1}{3}}\Gamma(\frac{4}{3})} \frac{\rho_{\mathrm{pw}}}{\rho_{\mathrm{p}\infty}} \frac{\psi(\theta)}{\psi(0)} \tag{30}$$

(where j'' is the local particle mass flux to the sphere: $j'' = -D\partial\rho_{\mathrm{p}}/\partial r|_{r=R}$), and also the overall deposition rate by integrating j'' over the sphere's surface. Defining

$$I(St) \equiv \int_0^\pi \frac{\psi(\theta, St)}{\psi(0, St)} \sin\theta \, d\theta, \tag{31}$$

then the total particle mass flux is

$$\dot{m} = 2\pi R^2 \rho_{\mathrm{p}\infty} U_\infty \frac{Pe^{\frac{2}{3}}}{2^{\frac{1}{3}}\Gamma(\frac{4}{3})} \frac{\rho_{\mathrm{pw}}}{\rho_{\mathrm{p}\infty}} I(St), \tag{32}$$

where values of I for selected Stokes numbers are given in table 1, and $\rho_{\mathrm{pw}}/\rho_{\mathrm{p}\infty}$ is the particle inertial enrichment just at the outer edge of the Brownian sublayer at the stagnation point. The factor I accounts for centrifugation effects, which tend to reduce $\dot{m}$ with increasing Stokes number, while the factor $\rho_{\mathrm{pw}}/\rho_{\mathrm{p}\infty}$ increases with St, and is calculated in §3.4.

I	1·400	1·2201	0·735	0·4612	0·2513	0·2127
St	0	0·05	0·25	0·50	1·0	1·2

TABLE 1

3.4. *Enrichment of the particle concentration along the stagnation trajectory for a sphere in Stokes flow: inertia-dominated region*†

The calculation of particle inertial build-up $\rho_{\mathrm{pw}}/\rho_{\mathrm{p}\infty}$ presents no difficulty because, away from the Brownian diffusion sublayer, the term $\mathbf{P}_{\mathrm{p}}$ may be neglected, and (4) and (5) constitute a totally hyperbolic system which may be integrated along the characteristics. The corresponding characteristic equations for the stagnation trajectory ($\theta = 0$) are (see appendix A)

$$\frac{d \ln \rho_{\mathrm{p}}}{dr} = -\left[u_r + \frac{2u}{r} + \frac{2v_\theta}{r}\right] \Big/ u, \tag{33}$$

$$u_r = \frac{U-u}{\tau u} = \frac{du}{dr}, \tag{34}$$

$$\frac{dv_\theta}{dr} = \left[\frac{V_\theta - v_\theta}{\tau} - \frac{v_\theta^2}{r} - \frac{v_\theta u}{r}\right] \Big/ u, \tag{35}$$

where for the purpose of simplifying the rather cumbersome notation we have used a new terminology defined in appendix A.

Equations (33)–(35) have to be integrated with the initial conditions

$$\rho_{\mathrm{p}} = \rho_{\mathrm{p}\infty}, \quad u = U, \quad v_\theta = V_\theta \quad (r \gg R),$$

† This subsection parallels our previous analysis (Fernández de la Mora & Rosner 1981) for the inertial enrichment of particles along the stagnation line of a cylinder at large Reynolds-number. Interestingly enough, none of the many previous particle-trajectory calculations (some older than one of us) have followed the evolution of ρ_{p}.

where U and V are given by the classical expressions (Landau & Lifshitz 1971)

$$U = U_\infty \cos\theta\left(1-\frac{3R}{2r}+\frac{R^3}{2r^3}\right),$$

$$V = -U_\infty \sin\theta\left(1-\frac{3R}{4r}-\frac{R^3}{4r^3}\right).$$

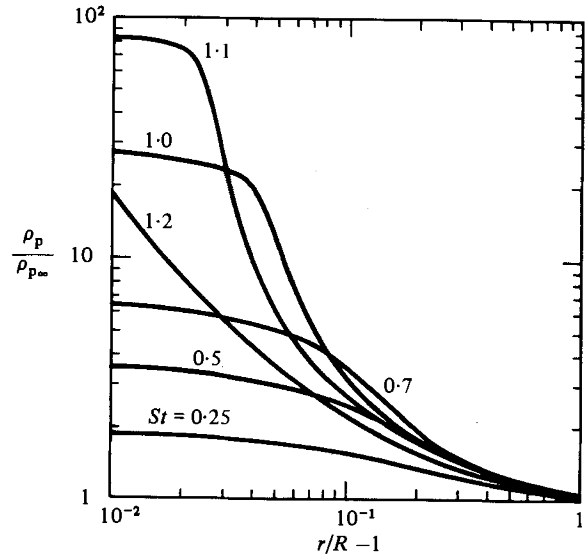

FIGURE 3. Inertial enrichment of local particle-phase density along the stagnation-point streamline at subcritical Stokes numbers.

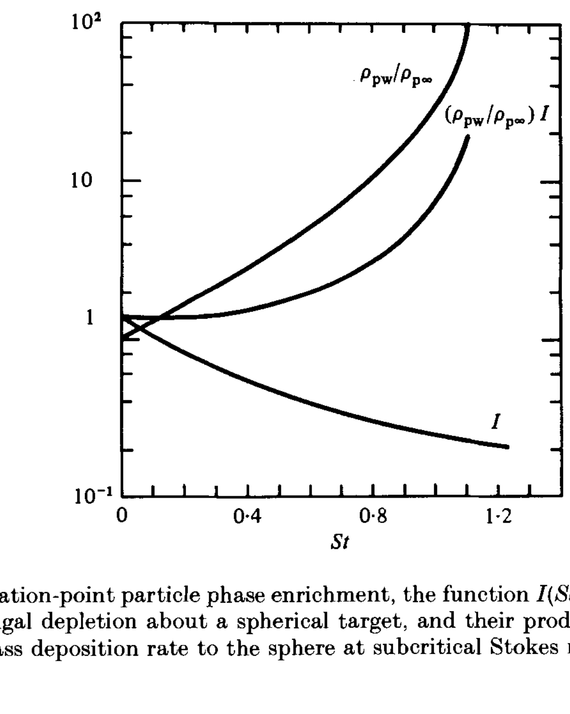

FIGURE 4. Stagnation-point particle phase enrichment, the function $I(St)$ governing the integrated effect of centrifugal depletion about a spherical target, and their product, which determines the total particle mass deposition rate to the sphere at subcritical Stokes numbers (cf. (32)).

The results are shown in figure 3 and 4. For very small values of the Stokes number the centrifugal depletion effect I dominates over the stagnation region enrichment $\rho_{\mathrm{pw}}/\rho_{\mathrm{p}\infty}$, and the corresponding overall particle capture rate is initially a *decreasing* function of St. Ultimately, however, this trend is reversed to the point that for $St = 1.1$, close to the critical Stokes number ($St_{\mathrm{c}} = 1{\cdot}21194$ according to Michael & Norey 1970) the inertial enhancement of the overall particle-deposition rate is larger than 10.

Michael & Norey (1970) gave an analytical expression for the first-order term of the particle-density expansion along the stagnation line in powers of the Stokes number. Their result may be rearranged as†

$$\ln\frac{\rho_{\mathrm{p}}}{\rho_{\infty}} = \frac{27}{8}\left[8\ln\left(1+\frac{1}{2r}\right)-\frac{4}{r}+\frac{1}{r^2}+\frac{1}{2r^4}\right]St+O(St^2), \tag{36}$$

and agrees well with our numerical calculations for values of the Stokes number of the order of, or smaller than, 0·3.

4. Conclusions

The transition between the diffusion- and inertia-dominated behaviour for a dilute suspension of small particles in a gas has been described in closed form via a hydrodynamic model. We have neglected diffusion effects except in the immediate vicinity of the solid body, where inertial effects have been included as a first-order perturbation term in the diffusion equation governing near-wall particle-phase behaviour. The model has been applied to the flow of a Newtonian incompressible host fluid about a sphere at low Reynolds numbers. The sphere's forward stagnation region is enriched in particles owing to compressibility effects in the particle phase, while downstream from the stagnation region inertial forces act in the opposite direction, tending to centrifuge the particles away from the wall, thus reducing local deposition rates by diffusion. We find that the overall rate of particle capture is significantly enhanced by inertia, except for a slight *reduction* in the region of Stokes numbers less than 0·3. The general formalism introduced may be generalized with minor changes to other geometries and other low-Reynolds-number flow models (such as the many used in filtration theory, which include rarefaction effects and the mutual interference between filter fibres). As an additional example, we sketch the case of Stokes flow about a cylinder in appendix B.

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Appendix A. Particle-phase compressibility along the stagnation trajectory of an axisymmetric body

The treatment here parallels our analysis (Fernández de la Mora & Rosner 1981) for the high-Reynolds-number motion of a dusty gas about a cylinder. To simplify the notation we adopt the following definitions with a totally different meaning than used previously:

$$u \equiv \text{radial particle-phase velocity} \quad (\text{previously } u_{\mathrm{p}r}),$$

† Note that r has been non-dimensionalized with respect to the sphere radius R.

$v \equiv$ angular particle-phase velocity (previously $u_{p\theta}$),
$U \equiv$ radial host-fluid velocity (previously U_r),
$V \equiv$ angular host-fluid velocity (previously U_θ).

The subindices r and θ now denote partial differentiation with respect to those variables: $u_\theta = \partial u/\partial \theta$, $U_r = \partial U/\partial r$, etc.... If the particle-phase pressure tensor is neglected, then the momentum-conservation equations (4) may be written in spherical coordinates as:

$$u u_r + \frac{v\, u_\theta}{r} - \frac{v^2}{r} = \frac{U-u}{\tau} \tag{A 1}$$

$$u\, v_r + \frac{v\, v_\theta}{r} + \frac{uv}{r} = \frac{V-v}{\tau}. \tag{A 2}$$

Taking derivatives with respect to r and θ in (A 1), we obtain

$$u u_{rr} + \frac{v\, u_{\theta r}}{r} = \frac{U_r - u_r}{\tau} - u_r^2 - u_\theta \frac{\partial (v/r)}{\partial r} + \frac{\partial (v^2/r)}{\partial r},$$

$$u u_{r\theta} + \frac{v\, u_{\theta\theta}}{r} = \frac{U_\theta - u_\theta}{\tau} - u_r u_\theta - u_\theta \frac{\partial (v/r)}{\partial \theta} + \frac{\partial (v^2/r)}{\partial \theta}.$$

Since it is also true that

$$du_r = u_{rr}\, dr + u_{r\theta}\, d\theta,$$

$$du_\theta = u_{r\theta}\, dr + u_{\theta\theta}\, d\theta,$$

we obtain, after regrouping in matrix form,

$$\begin{bmatrix} 1 & v/u & 0 \\ 0 & 1 & v/u \\ 1 & r\, d\theta/dr & 0 \\ 0 & 1 & r\, d\theta/dr \end{bmatrix} \begin{bmatrix} u_{rr} \\ u_{r\theta}/r \\ u_{\theta\theta}/r^2 \end{bmatrix} = \begin{bmatrix} \left[\dfrac{U_r - u_r}{\tau} - u_r^2 - u_\theta \dfrac{\partial}{\partial r}\left(\dfrac{v}{r}\right) + \dfrac{\partial}{\partial r}\left(\dfrac{v^2}{r}\right)\right]\Big/ u \\ \left[\dfrac{U_\theta - u_\theta}{\tau} - u_r u_\theta - u_\theta \dfrac{v_\theta}{r} + \dfrac{2 v v_\theta}{r}\right]\Big/ ur \\ du_r/dr \\ r^{-1}\, du_\theta/dr \end{bmatrix}.$$

Since along particle trajectories $v/u = r\, d\theta/dr$, the first two left-hand-side equations above are equal to the third and the fourth. Therefore the same has to hold for the right-hand-side, implying that for trajectories

$$\frac{du_r}{dr} = \left[\frac{U_r - u_r}{\tau} - u_r^2 - u_\theta \frac{\partial}{\partial r}\left(\frac{v}{r}\right) + \frac{\partial}{\partial r}\left(\frac{v^2}{r}\right)\right]\Big/ u,$$

$$\frac{du_\theta}{dr} = \left[\frac{U_\theta - u_\theta}{\tau} - u_r u_\theta - u_\theta \frac{v_\theta}{r} + \frac{2 v v_\theta}{r}\right]\Big/ u.$$

Analogously, taking derivatives in (A 2) we obtain

$$u\, v_{rr} + \frac{v v_{\theta r}}{r} = \frac{V_r - v_r}{\tau} - u_r v_r - v_\theta \frac{\partial}{\partial r}\left(\frac{v}{r}\right) - \frac{\partial}{\partial r} \frac{vu}{r},$$

$$u\, v_{r\theta} + \frac{v v_{\theta\theta}}{r} = \frac{V_\theta - v_\theta}{\tau} - u_\theta v_r - v_\theta \frac{\partial}{\partial \theta}\left(\frac{v}{r}\right) - \frac{\partial}{\partial \theta} \frac{vu}{r},$$

along with

$$dv_r = v_{rr}\,dr + v_{\theta r}\,d\theta,$$

$$dv_\theta = v_{r\theta}\,dr + v_{\theta\theta}\,d\theta,$$

which by an identical process with the one followed for the u-component yields finally

$$\frac{dv_r}{dr} = \left[\frac{V_r - v_r}{\tau} - u_r v_r - v_\theta \frac{\partial}{\partial r}\left(\frac{v}{r}\right) - \frac{\partial}{\partial r}\left(\frac{vu}{r}\right)\right]\Big/ u, \tag{A 3}$$

$$\frac{dv_\theta}{dr} = \left[\frac{V_\theta - v_\theta}{\tau} - u_\theta v_r - v_\theta \frac{\partial}{\partial \theta}\left(\frac{v}{r}\right) - \frac{\partial}{\partial \theta}\left(\frac{vu}{r}\right)\right]\Big/ u. \tag{A 4}$$

The mass-conservation equation for the particle phase is:

$$u\frac{\partial \rho_p}{\partial r} + \frac{v}{r}\frac{\partial \rho_p}{\partial \theta} = -\rho_p\left[\frac{1}{r^2}\frac{\partial}{\partial r}(ur^2) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v\sin\theta)\right]$$

$$= -\rho_p\left[u_r + \frac{2u}{r} + \frac{v_\theta}{r} + \frac{v\cos\theta}{r\sin\theta}\right]$$

where the term $v\cos\theta/r\sin\theta$ is indeterminate, but it is easily shown that because $v(\theta=0)=0$,

$$\lim_{\theta\to 0}\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v\sin\theta) = \frac{v_\theta}{r\theta}\frac{\partial\theta^2}{\partial\theta} = \frac{2v_\theta}{r}.$$

Thus along the stagnation line

$$\frac{d\ln\rho_p}{dr} = -\left[u_r + \frac{2u}{r} + \frac{2v_\theta}{r}\right]\Big/ u, \tag{A 5}$$

$$\frac{du}{dr} = \frac{U-u}{\tau u}, \tag{A 6}$$

$$\frac{du_r}{dr} = \left[\frac{U_r - u_r}{\tau} - u_r^2\right]\Big/ u, \tag{A 7}$$

$$\frac{dv_\theta}{dr} = \left[\frac{V_\theta - v_\theta}{\tau} - \frac{v_\theta^2}{r} - \frac{v_\theta u}{r}\right]\Big/ u, \tag{A 8}$$

which may be easily integrated given the appropriate boundary conditions.

Appendix B. Inertial and diffusive deposition of particles on a cylinder in low-Reynolds-number flow

Near the cylinder's surface, the host-fluid velocity field is given by

$$\frac{U_r}{U_\infty} = -\tfrac{1}{2}\eta^2 C\cos\theta, \tag{B 1}$$

$$\frac{U_\theta}{U_\infty} = \eta C\sin\theta, \tag{B 2}$$

where η is defined by (11) when R is the cylinder radius and C is a function of the Reynolds number which for the particular case of the Stokes–Oseen flow takes the form (see Batchelor 1967)

$$C = \frac{2}{\log(7\cdot 4/R)}. \tag{B 3}$$

Analogously, as in §3.3, the particle mass-conservation equation takes the form (we now adopt the new definition $St = CU_\infty \tau/R$)

$$(CPe)^{-1}\frac{\partial^2 \rho_p}{\partial \eta^2}+\eta^2(\tfrac{1}{2}\cos\theta-\sin^2\theta\, St)\frac{\partial \rho_p}{\partial \eta}-\eta\sin\theta\frac{\partial \rho_p}{\partial \theta} = 2St\,\eta\sin^2\theta\rho_p \tag{B 4}$$

and calculation of the subcharacteristics suggests the use of the new variables

$$\xi = \eta\sin^{\frac{1}{2}}\theta\exp(St\cos\theta), \tag{B 5}$$

$$n = \rho_p\exp(-2St\cos\theta) \tag{B 6}$$

(leaving θ unchanged), in terms of which the mass-conservation equation reduces to

$$\xi^{-1}\frac{\partial^2 n}{\partial \xi^2} = Pe\,C\exp(-3St\cos\theta)\sin^{\frac{1}{2}}\theta\frac{\partial n}{\partial \theta}, \tag{B 7}$$

and finally to the familiar Leveque form

$$\xi^{-1}\frac{\partial^2 n}{\partial \xi^2} = Pe\,C\frac{\partial n}{\partial s}, \tag{B 8}$$

after defining

$$s \equiv \int_0^\theta \sin^{\frac{1}{2}}\theta'\exp(3St\cos\theta')\,d\theta'. \tag{B 9}$$

The problem is thus equivalent to the one solved in §3 except for the slightly different definitions of the variables n, ξ and s.

Appendix C. Outer boundary condition for the diffusion boundary layer

From (27′) the thickness of the diffusion boundary layer can be chosen such that $\frac{1}{9}\mu^3 \approx 1$. The characteristic trajectory defining the boundary of the streamtube feeding the diffusion boundary layer can thus be chosen such that it is tangent to the line $\mu = \text{const} = 9^{\frac{1}{3}}$:

$$\text{characteristic:}\quad \eta_c = \frac{\xi_*}{\sin\theta}\exp(-\tfrac{3}{2}St\cos\theta); \tag{C 1}$$

$$\mu = \text{const:}\quad \eta_\mu = \mu(\tfrac{4}{3}Pe)^{-\frac{1}{3}}\frac{s^{\frac{1}{3}}}{\sin\theta}\exp(-\tfrac{3}{2}St\cos\theta). \tag{C 2}$$

The tangency condition requires

$$\eta_c = \eta_\mu, \tag{C 3}$$

$$\left.\frac{d\eta_c}{d\theta}\right|_{\xi_*} = \left.\frac{d\eta_\mu}{d\theta}\right|_\mu. \tag{C 4}$$

Equation (C 3) is equivalent to (27″), while (C 4) implies $ds/d\theta = 0$, or $\theta = \pi$ (the solution $s = 0$ is incompatible with (C 3)). The two conditions clearly fix ξ_* and θ, the former determining the boundary of the streamtube of interest. It remains to calculate $s(\pi)$, though, instead, we shall use the following bound (equivalent to $\cos\theta \leqslant 1$ for $0 \leqslant \theta \leqslant \pi$):

$$s(\pi) < \tfrac{1}{2}\pi\exp(\tfrac{9}{2}St). \tag{C 5}$$

Thus from (27″) ξ_* is bounded by

$$\xi_* < \mu_*\left(\frac{4Pe}{3\pi}\right)^{-\frac{1}{3}}\exp(\tfrac{3}{2}St), \tag{C 6}$$

or, after taking $\mu_* = 9^{\frac{1}{3}}$ and using (22),

$$\eta_* < \left(\frac{4Pe}{27\pi}\right)^{-\frac{1}{3}} \frac{1}{\sin\theta} \exp\left[\tfrac{3}{2} St(1-\cos\theta)\right]. \quad \text{(C 7)}$$

Close to the stagnation point $\theta \approx 0$, and (C 7) may be rewritten as

$$\theta\eta_* < \left(\frac{4Pe}{27\pi}\right)^{-\frac{1}{3}}.$$

This trajectory is a hyperbola in the (θ, η)-plane, and when $Pe \gg 1$ it clearly approaches the singular hyperbola $\theta\eta_* = 0$ (the stagnation streamline). Furthermore, as may be seen in figure 2, there is a region of thickness of the order of $\eta \approx 0{\cdot}1$ below which the effect of particle phase compressibility dies away (clearly this is no longer true when $St \gtrsim 1.1$). At the outer edge of this region one has that $\theta \approx Pe^{-\frac{1}{3}}$, and, as anticipated, the streamtube of capturable particles is indeed rather small.

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Note added in proof

After this paper had been accepted for publication, J. D. Ramshaw sent us his papers: Ramshaw (1979), 'Brownian motion in a flowing fluid', *Phys. Fluids* **22**, 1595–1601 and Ramshaw (1981), 'Brownian motion in a flowing fluid revisited', *Phys. Fluids* **24**, 1210–1211. In these papers our equations (4)–(6) have been presented as a closed system describing the particle motion, and including a fluid–particle interaction force which is more general than ours (see our equation (7)) and which accounts for the Archimedean lift and thermophoresis. The author has also shown that pressure diffusion results from the particle inertia. However, some of his generalizations are inconsistent: if the thermophoretic force is included in the presence of temperature gradients, then the system is no longer in thermal equilibrium and the ideal gas law is no longer exact. Equation (18) of Ramshaw (1979) is thus invalid. The same inconsistency arises in the presence of velocity gradients, and Ramshaw does not mention that the particle pressure is given correctly by the ideal gas law only for small Stokes numbers. But then it is not necessary to solve (4)–(6), since $\mathbf{V}_P$ is given by (9).

John Fenn has referred us to the early work of H. Thoman, 'Determination of the size of ice crystals formed during condensation of water in wind tunnels and of their effects in boundary layers' (*Report* 101 *of the Aeronautical Research Institute of Sweden, Stockholm, July* 1964). There, the close connection between particle inertia and pressure diffusion was implicit already (Appendix B).

Finally, G. K. Batchelor has independently presented closed equations equivalent to our equations (4)–(6), and has used them to show the influence of particle inertia on the stability of fluidized beds of small particles (G. K. Batchelor, 'Sedimentation of small particles', paper presented at the 4th International Conference on Physico–Chemical Hydrodynamics, The City College, New York, 13–17 June 1982).